

Les diagonales d'un polygone

Niveau : 5ème

Objectif : Introduire le calcul littéral

Durée estimée de la totalité de l'activité : 1h30

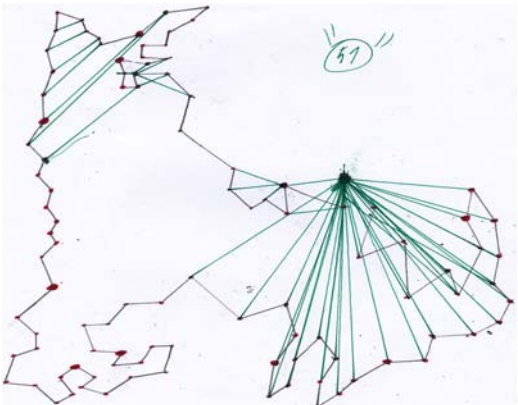
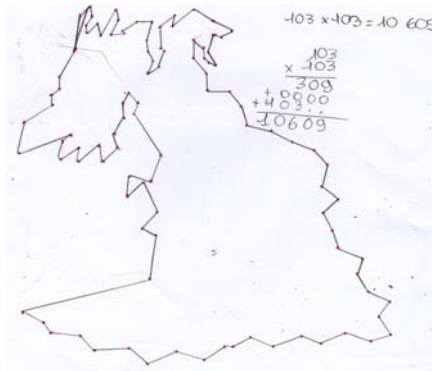
Énoncé :

Un quadrilatère possède 2 diagonales.
 Un pentagone possède 5 diagonales.
 Combien de diagonales un polygone à 103 côtés possède-t-il ?

Modalités de gestion possibles :

- recherche individuelle (15 minutes)
- travail par groupes d'élèves réunis par « stratégies » (45 minutes)
- mise en commun ; débat sur les différentes stratégies (30 minutes)

Quelques productions individuelles d'élèves (« premiers jets »)

																																																																																																																														
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